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Effect of Boundary Conditions on Free Vibration Analysis of Thick Fiber Reinforced Plastic Skew Laminate with Circular Cutout

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Abstract

The present investigation deals with the free vibration analysis of a thick four-layered skew laminated composite plate with a circular cutout. Three dimensional finite element models (FEM) which use the elasticity theory for the determination of stiffness matrices are modeled in ANSYS software to evaluate first five natural frequencies of the laminate. The variation of natural frequencies with respect to various boundary conditions for different stacking sequences is presented. It is observed that, the natural frequencies are high at c-c-c-c (four sides clamped) case.

Keywords: Fiber reinforced plastic (FRP), finite element model (FEM), skew plate, cutout, natural frequency, free vibration.

Introduction

Composite materials are replacing metals in many structural applications such as aerospace, transportation, novel and pressure vessels due to their high specific strength and specific modulus. The dynamic responses of these structures differ from that made of isotropic materials due to orthotropic nature of individual layers of the laminate. Free vibration analysis is a part of dynamic analysis where the natural frequencies of the structure can be estimated. The following paragraph provides a brief review of various research contributions on free vibration analysis of Fiber reinforced plastic (FRP) composite structures. Based on a higher order shear deformation theory, Ajay et al. [1] presented the free vibration analysis of sandwich skew laminates. The three-dimensional theory of laminated plates and shells has been developed by Chao and Yeong-Chyuan [3] for the free vibration frequencies of rectangular plates. Isaac et al. [7] described the concepts of micro and macro mechanics of the

composite materials in detail. The free vibrations of multi layered composite plates are studied by Noor [8]. An eight-noded hybrid-stress finite element is used by Ramakrishna et al. [10] to analyse the dynamic behavior of laminated plates with cutouts. Free vibration frequencies of symmetric and anti-symmetric laminated plates are studied by Kant and Swaminathan [4], Panda and Singh [9], Viswanathan and Lee [12] using higher order shear deformation theories. The vibration analysis of angle-ply laminated beams subjected to different sets of boundary conditions is investigated by Bahar and Metin [2], Li and Hua [5], Metuin [6] and Taehyo et al. [11] studied linear static and dynamic analysis of laminated composite plates and shells using a 4-node quasi-conforming shell element. The present investigation intends to apply three-dimensional finite element techniques for the free vibration analysis of thick skew laminates with circular cutout.

Problem statement

A. Geometric modeling

The geometry of the problem is shown in Figure 1. The sides of the plate 'l' and 'b' are taken equal to 2 m and the height of the plate is divided in to four layers of equal thickness ($h/4$), Where 'h' is the total thickness of the laminate. The skew angle (α) is taken as 30° . The circular hole is placed at the geometric centre of the plate. The diameter of the hole is taken as per the d/l ratio of 0.3, the length-to-thickness ratio is taken 10.

B. Finite Element Modeling

The element used for the present analysis is SOLID 95 of ANSYS, which is developed, based on three-dimensional elasticity theory and is defined by 20 nodes having three degrees of freedom at each node, translation in the node x, y and z directions. The model with finite element mesh is shown in Figure2.

C. Boundary Conditions

The sides of the skew laminate considered for the present analysis are clamped that is, all the three degrees of freedom of the nodes along the four sides of the skew plate are constrained.

D. Material properties

Each layer is unidirectional carbon fiber reinforced plastic possessing the following engineering constants Ramakrishna et al. (1993):

- (i) Elastic modulus in the longitudinal direction of the fiber, $E_L = 175$ GPa,
- (ii) Elastic modulus in the transverse direction of the fiber, $E_T = 7$ GPa,
- (iii) Shear modulus in the longitudinal plane of the fiber, $G_{LT} = 3.5$ GPa,
- (iv) Shear modulus in the transverse plane of the fiber, $G_{TT} = 1.4$ GPa,
- (v) Poisons ratio, $\nu_{LT} = \nu_{TT} = 0.25$,
- (vi) Mass density, $\rho = 1.6 \times 10^3$ kg/m³

E. Validation of finite element model (FEM)

The 3-D FEM is validated with the results available in the literature and found good agreement (Table 1).

F. Stacking sequences

Two or more layers of the composite material are bonded together to form a laminated composite structure in order to obtain tailored properties in required direction. The indication of fiber position in each lamina of the laminated composite is designated by their stacking sequence. In continuation of finite element modeling, Figure3

show the details of layer arrangement in four layered symmetric cross-ply laminated composite plate of angle $0/90/90/0$. From the side view of these figures, the circle represents zoomed view of the element coordinate system (ESYS). In the element coordinate system, the co-ordinate 11 represents the 0° angle and the co-ordinate 12 represents the 90° angle.

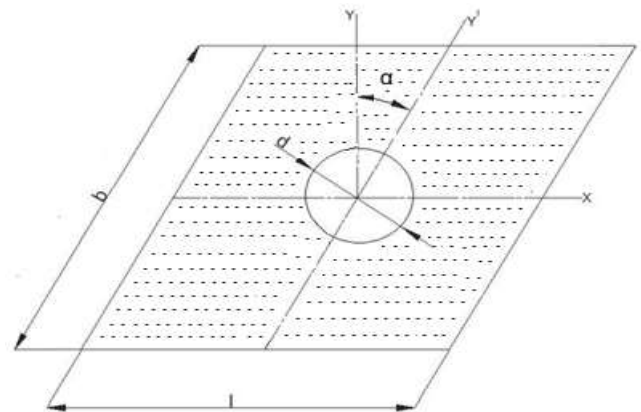


Figure 1. Skew plate with cutout

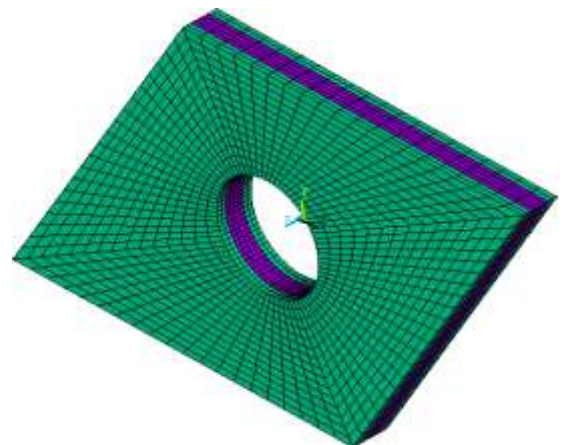


Figure 2. Finite Element mesh on skew plate

Effect of support conditions

A. Influence of boundary conditions on cross-ply laminate

The variation of first five natural frequencies with respect to boundary conditions on a cross-ply laminate is shown in Figure 4. In that, taken lamination scheme is $0/90/90/0$. The geometric parameters are taken $d/l=0.3$, $\alpha=30^\circ$, $s=10$ and $n=4$. The different combinations of simply supported and clamped boundary conditions are considered as s-s-s-s, s-c-s-s, s-s-c-s, s-s-c-c, c-s-s-s, s-s-s-c, s-c-c-s, c-s-c-s, s-c-s-c, c-s-s-c, c-s-c-c, c-c-s-s, s-c-c-c,

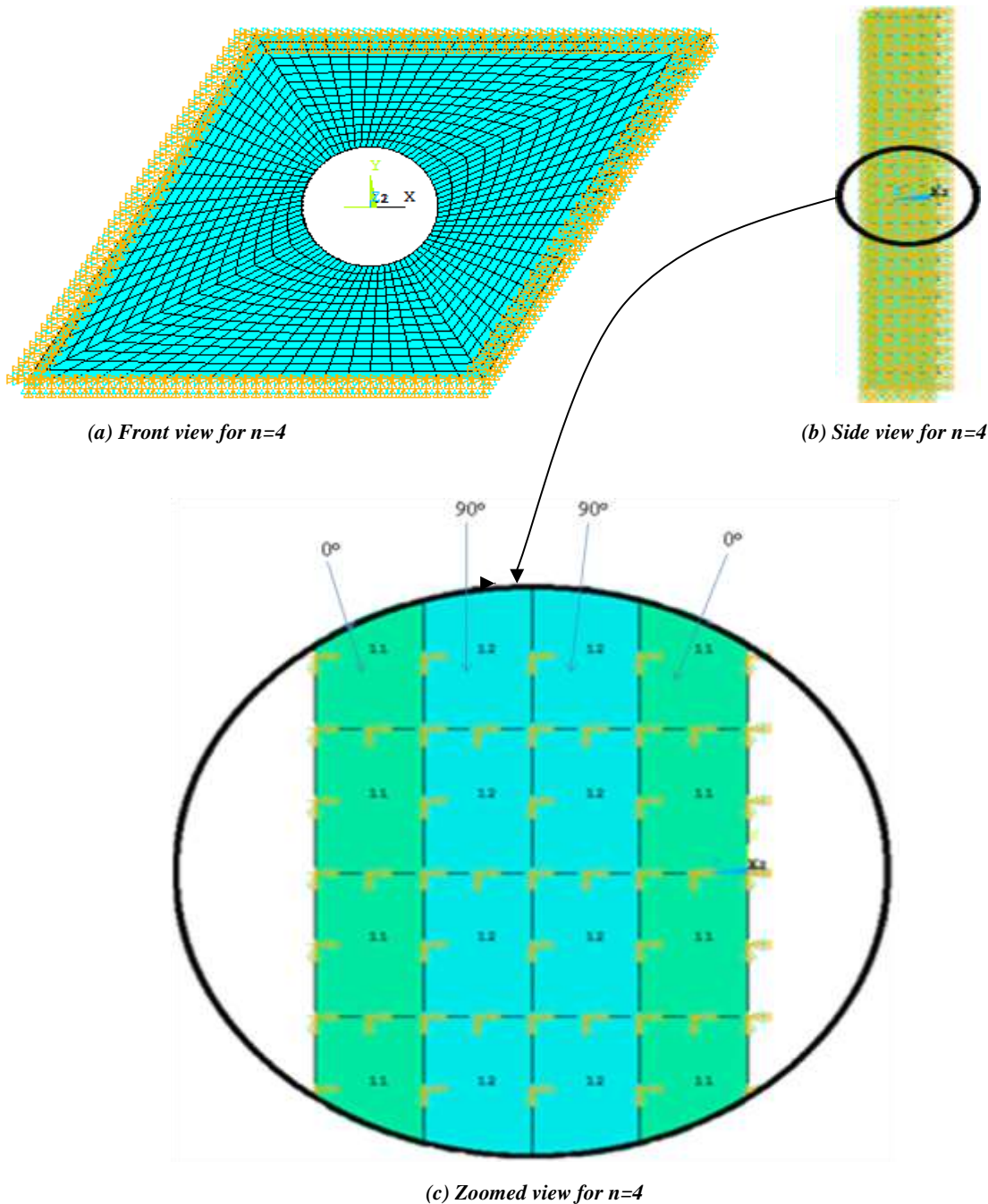


Figure 3. Four layered Laminate showing details of layer arrangement

Table 1. Validation of the finite element results using the 3-D elasticity theory for Ω_c , Ω_d

Thickness ratio (a/h)	a/h=5	a/h=5
Sliding pin supported edge (S_2)	$[0/90 \dots]_6^i$	$[45/-45]_{ii}$
Present (3-D FEM)	3.4088	8.5707
Chao (2000)	3.3495	8.2872
Noor (1973)	3.4250	---

Where the normalized frequency,

$$\Omega_c = 10\omega h(\rho_m/E_2)^{1/2} \text{ and } \Omega_d = \omega a^2(\rho_m/E_2 h^2)^{1/2}$$

c-c-c-s, c-c-c-c. In all cases of boundary conditions, the natural frequency increases with respect to increase in the mode number. By comparing all modes of boundary conditions, the frequencies are lower at s-s-s-s boundary condition and the rate of increment in natural frequency is high at c-c-c-c boundary condition because of its constraints on the degree of freedom. Among all these boundary conditions, c-c-c-c boundary condition shows more resistance to deformation, which in turn increasing the natural frequency and resulting increase in stiffness of the structure.

B. Influence of boundary conditions on angle-ply laminate

The variation of first five natural frequencies with respect to various boundary conditions at different modes of angle-ply laminate is shown in Table 2. The geometric parameters are considered $d/l=0.3$, $\alpha=30^\circ$, $s=10$ and $n=4$. In that, taken lamination sequence is 30/-30/-30/30. From the Table 2, the variation of natural frequency gradually increases with respect to increase in mode number and also by comparing the boundary conditions with respect to mode number, the frequencies are lower at s-s-s-s boundary condition and higher at c-c-c-c boundary condition. It is observed that, the frequency is increasing due to reduction of mass than stiffness. The arrangement of contour plots for the first five modes of natural frequencies on s-s-s-s and c-c-c-c boundary conditions is shown in Figures 5 and 6.

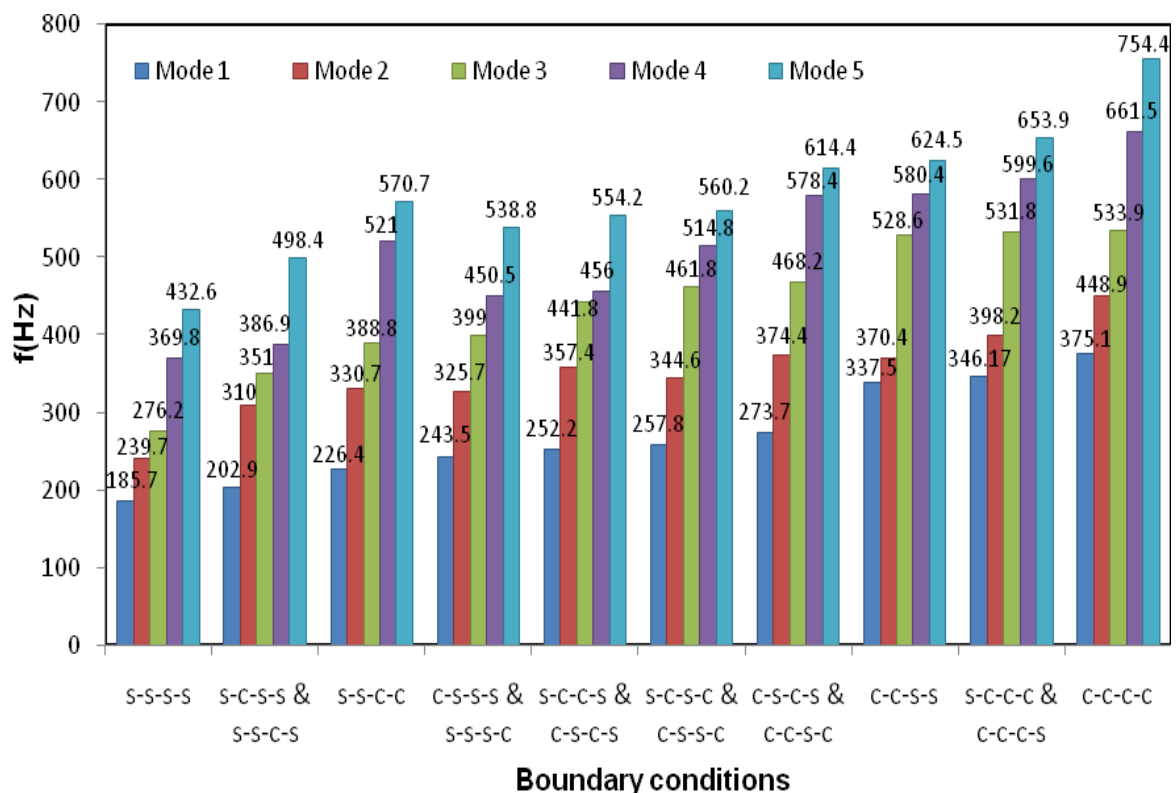


Figure 4. Variation of natural frequency with respect to boundary conditions for a lamination scheme of 0/90/90/0

Table 2. Variation of natural frequency with respect to boundary conditions for a lamination sequence of 30/-30/-30/30

Boundary conditions	Mode1	Mode2	Mode3	Mode4	Mode5
s-s-s-s	Minimum 228.8	262.29	336.42	399.42	504.76
s-c-s-s & s-s-c-s	241.26	354.23	381.4	407.59	521.11
s-s-c-c	257.88	366.83	419.45	538.56	640.46
c-s-s-s & s-s-s-c	275.43	368.89	471.87	515.01	557.91
s-c-c-s & c-s-c-s	285.04	393.46	474.4	552.66	566.62
s-c-s-c & c-s-s-c	285.76	385.71	482.54	576.63	662.06
c-s-c-s & c-c-s-c	298.91	408.33	484.64	586.17	673.61
c-c-s-s	358.03	405.24	534.67	610.44	693.61
s-c-c-c & c-c-c-s	366.92	425.62	537.56	622.33	703.14
c-c-c-c	Maximum 387.21	446.72	559.81	640.29	717.34

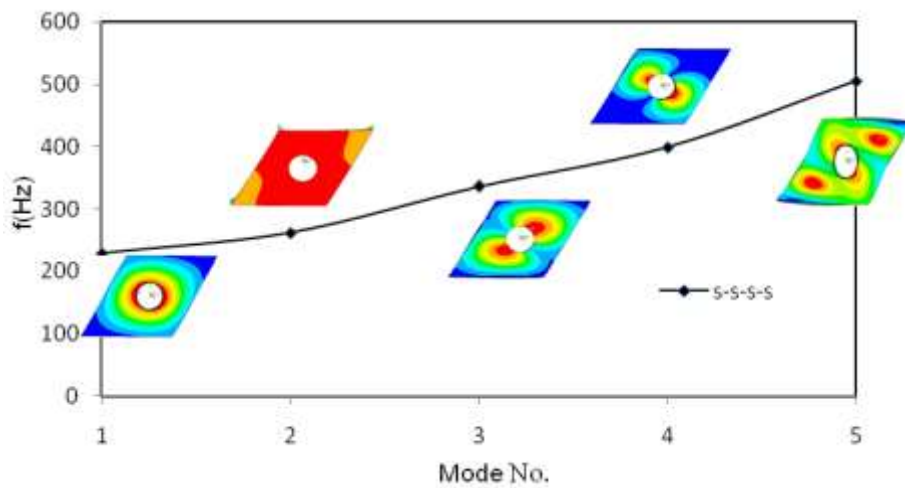


Figure 5. Variation of 'f' on boundary condition (s-s-s-s) for a sequence of 30/-30/-30/30

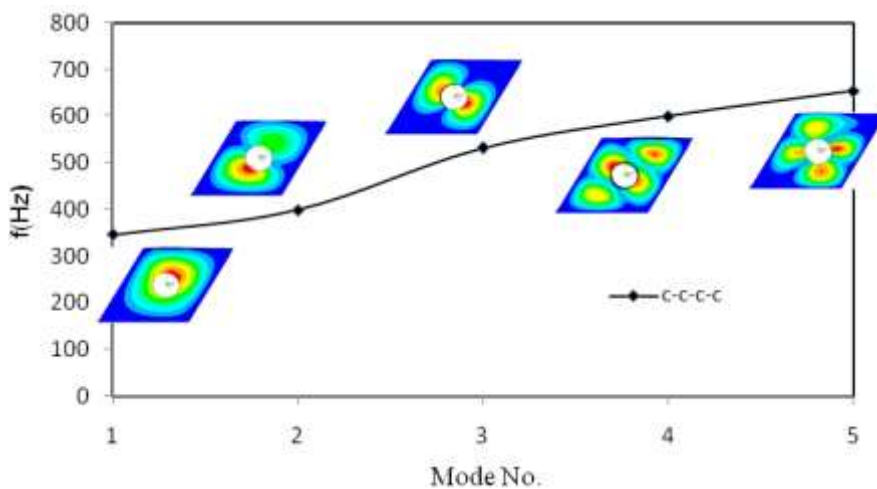


Figure 6. Variation of 'f' on boundary condition (c-c-c-c) for a sequence of 30/-30/-30/30

The variation of 'f' with different lamination schemes of boundary conditions for the first mode results is presented. From the Figure 4, the minimum frequency is observed at (s-s-s-s) boundary condition and maximum frequency is observed at (c-c-c-c) boundary condition in (0/90/90/0) lamination scheme. From the Table 2, the minimum frequency is observed at (s-s-s-s) and maximum frequency is observed at (c-c-c-c) in (30/-30/-30/30) stacking sequence.

Conclusion

Free vibration analysis of a thick four-layered skew laminated composite plate with a circular cutout has been taken up in the present work. From the effect of various boundary conditions, the minimum frequency is obtained at s-s-s-s boundary condition and the maximum natural frequency is obtained at c-c-c-c boundary condition in (0/90/90/0) lamination scheme. The present analysis is useful for the design of skew plates with cutouts for dynamic response.

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